Optimal Maintenance and Replacement Decisions under Technological Change

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ABSTRACT: The requirement of equipment improvement in order to satisfy the safety and reliability of system motivates the development of technology. The presence or expectation of technologically better equipment will influence managerial decisions on whether to invest in the maintenance of current equipment, invest in replacement with an equivalent model, replacement with a higher technology model currently available on the market, or wait for a potentially even better technology to appear in the near future. Hence, the consideration of technological change is a very important aspect for maintenance and replacement decisions. This paper aims to define a model that allows us to gain insight into how maintenance/replacement policies will be influenced by the expectation of future technology. We then use stochastic dynamic programming (i.e., Markov decision process) to solve for the optimal maintenance and replacement policy of the equipment as a function of performance and cost. Finally, we illustrate the problem through several numerical examples.

1 INTRODUCTION

One of the most important objectives for the maintenance managers is to balance the maintenance action costs with the investment cost. There is an intensive research to provide the most appropriate strategies for organizing a set of maintenance actions generally based on complex degradation models to optimize a decision criterion. These models usually consider different maintenance actions from the "as good as new" replacement by an identical item, imperfect maintenance which restores the item to an acceptable condition and minimal or "as bad as old" repair. They do not allow us to take into account the appearance of new technology with lower operating and maintenance costs, smaller failure rates, higher quality and output rates. This information is important for managers to decide the replacement investment plan. On the other side, the models devoted to the optimization of the investment planning with the introduction of the technological change (TC) are generally based on the observations of the economical performance of the maintenance process. The proposed models do not consider the diversity of maintenance decisions tackled by the deterioration-based maintenance models.

These reasons motivate us to provide an appropriate model to meet the operational and strategic requirements. This model is developed to organize both the maintenance and replacement decisions for a continuously deterioration system, under technological evolution.

Most of the references considering the influence of technology change are strictly economical-oriented (Bethuyne 2002, Bean et al. 1994, Elton et al. 1976, Goldstein et al. 1986, Goldstein & Mehrez 1996, Hritoneko & Yatsenko 2007, 2008a, b, 2009, Huisman & Kort 2004Nair 1995, 1997, Ott et al. 1995, Smith et al. 2003, 2007, Rajagopalan 1999). Their approach are based more on the modeling of the maintenance process through cost functions such as evolution of the operating-maintenance cost instead of the traditional failure indicators such as the degradation or failure rate models. Thanks to these models, the managers can decide the best time for replacement investment of equipment under technological evolution but do not consider the maintenance strategies as well as the impact of technology change on it.

The present work continues in the direction introduced by articles (Borgonovo et al. 2000, Clavareau & Labeau 2009a, b, Dogramaci & Fraiman 2004, Hopp & Nair 1994, Karsak & Tolga 1998, Michel et al. 2004, Mercier 2008) that take into account the failure characteristics of the equipment in the replacement problem under technological development. We construct a model that allows us to decide whether to imperfectly maintain, to replace with a higher technology model currently available on the market, or wait for a potentially even better technology to appear in the near future. The technology evolution is modeled by a non-homogenous Poisson process and the effect of a new technology is measurable through the degradation characteristics. We also consider that performance of equipment in use will stochastically degrade over time due to deterioration while its accrued profit and maintenance cost will stochastically decrease and increase respectively. The objective of this paper is to maximize revenue over a given planning period horizon. The revenue is defined here as the difference of the incomes and the outcomes. The incomes are the profit, a function of the associated degradation level, and the salvage value in case of replacement which is increasing in the expected mean residual life and proportional to the purchase price of new identical item. The outcomes are the different maintenance costs and the purchase price in case of replacement with a new technology. A discrete time non-stationary Markov Decision Process (MDP) formulation is proposed to determine the optimal action plan.

This paper is structured as follows: In Section 2, a related literature is presented to motivate the present work. Section 3 is devoted to the mathematical formulation. In Section 4, the performance of our model is discussed through numerical examples. Finally, a conclusion and future work are discussed in Section 5.

2 RELATED LITERATURE

There are few articles that consider the maintenance – replacement problem under technological development with degradation performance. The articles of Clavareau & Labeau (2009a, b), Michel et al. (2004), Mercier (2008) examine preventive, corrective replacement strategies of N identical components. However, because of the complexity of the system, they must simplify the technological evolution model. They consider a single new technology that has already appeared on the market with deterministic parameters. These assumptions are very limited because the technology develops rapidly and continuously. In addition, they are interested only in finding the optimal policy to replace obsolete equipments by new type, without considering whether the replacements is necessary. In our model, to attach special importance to both the continuation and the flexibility of technological change, we simplify the model by examining single equipment such as articles (Borgonovo et al. 2000, Hopp & Nair 1994, Dogramaci & Fraiman 2004).

We formulate a discrete time non-stationary Markov Decision Process to determine the optimal maintenance – replacement policy. The similar model with non-stationary technological appearance's probability in time was proposed by Nair (1995, 1997). However, in these papers, the author only considers replacement problem, not examining decision to maintenance as our model. Karsark & Tolga (1998) integrate overhaul policy into replacement problem. With geometric technological evolution model, they formulate a discrete time Markov decision process to determine an optimal overhaul-replacement policy which maximizes the expected present worth over a finite horizon time. But they also study this problem from manager's point of view, not taking into account failure rate or deterioration process of machines. Considering of parametric performance represented by a Markovian deterioration process, Hopp & Nair (1994) also utilize MDP algorithm to dealing the equipment replacement problem under technological change. However, recall that they only consider the equipment replacement problem while our work also examines the decision of maintenance. On the other hand, unlike Hopp & Nair (1994) reviewing unique challenger, we study a technological sequence.

To model sequential technological evolution, we combine the geometric model and uncertain apparition model of technology. The geometric technological evolution model is presented by Borgonovo et al. (2000), Smith et al. (2003), Karsak & Tolga (1998), Hritoneko & Yatsenko (2007, 2008a, b), Bethuyne (2002). But except Borgonovo et al. (2000), the rest study the problem without parametric degradation. They utilize the geometric model to form the cost functions in vintage equipment or in time. Unlike these articles, we present technology change by the improvement of the expected deterioration rate. Moreover, our profit or maintenance cost functions are only dependent on degradation state. As the expected degradation rate of equipments is improved over technology generation, accrued profit and maintenance cost will be dependent on technology generation.

In addition, we also consider non-stationary likelihood of new technology's apparition over time. Thereby, we overcome the disadvantages of the geometric model proposed by Borgonovo et al. (2000). In that article, the failure rate decreases exponentially over time, i.e. at any time, a machine can be replaced by new one which operates better with its reliability parameters determined at that time. In reality, this assumption is unreasonable because technical characteristics of the equipment can't always be changed over time. It changes only at the concurrent instant of a new technological generation. Recall that Nair (1995, 1997) also considers the non-stationary probability of the appearance of new technologies. But in his model, Nair focuses on the problem of capital investment decisions due to technological change rather than physical deterioration of equipment. To simplify its exposition, he also don't consider salvage values while we establish the reasonable salvage value function which depends on its mean residual life and the purchase price of identical technology at this time.

3 MODEL FORMULATION

3.1 *Maintenance problem*

Consider a repairable machine that operates continuously from the new state, $X = 0$, until a failure threshold, ζ. A machine is characterized by its expected deterioration rate. In the failure state, denoted m, the machine continues to operate but unprofitably. To reveal the deterioration level, periodic inspections are performed. The inter-inspection interval, τ, defines the decision epochs.

We assume that only one new technology can appear in a decision interval, τ. We introduce p_{i+1}^{k+1}, the non stationary probability that technology k+1 appears in the interval τ given the latest available technology at decision epoch i is k. The difference in the technologies k and k+1 is modeled by an improvement factor on the expected instantaneous deterioration rates.

Let (x, k, j) be the system state at the beginning of the i^{th} decision epoch with observed deterioration level x when technology j is used and the technology k, $k \geq j$, is available. Then, the maintenance decisions are restricted to:

1) Do nothing (*DN*): The machine continues to deteriorate until next decision epoch and generates a profit $g(x)$. Note that $g(x)$ is the accrued profit within a period, depends only on the deterioration state at the beginning of that period. This assumption is not very restrictive in case where the decision period is sufficiently small and the decreasing rate of the profit function in deterioration state is not very fast.

2) Maintain (*M*) which allows to restore the machine in a given deterioration level, max(0, x-d) where d models the maintenance efficiency. An increasing maintenance cost in the deterioration, $c_M(x)$, is incurred and as we assume that the maintenance time is negligible, then, in the next decision interval, the machine deteriorates from the level x-d and generates a profit $g(x$-$d)$.

3) Replace (*R*) the equipment with the latest available technology k. The replacement time is also negligible. The cost of such a replacement is given by the difference between the purchase price of the new machine $c_{i,k}$ and the salvage value $b_{i,j}(x)$. The purchase price is an increasing function of technology and decreasing over time. The salvage value is proportional to the purchase price of technology j and decreasing in the remaining lifetime. In the i^{th} decision period after the replacement, the new machine generates a profit $g(0)$. Note that as the deterioration rates of new technological machine, the purchase price can be estimated. This is realistic in case where the technical parameters and specifications of futures designs may be know beforehand.

In case of failure, the do nothing action is still allowed but the profit in the next decision epoch is assumed to be negative $g(m) < 0$.

3.2 *Decision criteria formulation*

In this paper, we use a non-stationary MDP formulation to find the optimal maintenance-replacement policy to maximize the expected discounted value-to-go over the finite horizon time denoted by $V^{\pi}(s)$. If the last decision period is N, at decision epoch N+1, we do not make any decision and the maximum expected discounted value-to-go from the decision epoch N+1 over the infinite horizon is $V_{N+1}(s)$ = 0 ($\forall s \in S$: state space of system).

Let $V_i(x,k,j)$ denote the maximum expected discounted value from the decision epoch i, ($k \leq i$) to the last epoch N. Then, $V_1(s) = V^{\pi}(s)$.

$$V_i(x,k,j) = \max\{DN_i(x,k,j),\ DM_i(x,k,j),\ R_i(x,k,j)\} \tag{1}$$

where DN_i, M_i, R_i are alternately choice to do nothing, to maintain and to replace at decision epoch i^{th}. We have:

$$DN_i(x,k,j) = g(x) + \lambda[\sum_{\forall k' \in \{k,k+1)} \sum_{\forall x' \in [x,\zeta]} p_i(x',k',j \mid x,k,j) V_{i+1}(x',k',j)] \tag{2}$$

$$M_i(x,k,j) = -c_M(x) + DN_i(\max\{0, x-d\}, k, j) \tag{3}$$

$$R_i(x,k,j) = -c_{i,k} + b_{i,j}(x) + DN_i(0,k,k) \tag{4}$$

λ: discount factor; $\lambda \in [0, 1]$.

3.3 *Transition probabilities*

To compute the transition probabilities, we propose to discretize the deteriorating state of machine as follows. Let z denote the discrete deterioration state at the beginning of the current decision period. z is the first value of N_X discrete intervals of length l on [0, ζ] (which ζ is the failure threshold of the machine). That is to say, if the deterioration state (x) at the beginning of current decision period belongs to the intervals ([0, l[, [l, 2l[, [2l, 3l[, ..., [(N_X - 1)l, ζ[), we approximate x by $z \in \{0,\ l,\ 2l,\ 3l\ \ldots\ (N_X\text{-}1)l\}$ and when the deterioration state (x) at the beginning of current decision period is exceed the failure threshold ($x \geq \zeta$), we use m to present failure state of the machine. The deterioration state of the machine (x) is approximated by z, $z \in \{0, l, 2l, 3l \dots (N_X\text{-}1)l, m\}$. Then, after preventive maintenance, the deterioration state is reduced by a determined amount of deterioration units d. The transition probability is:

$$p_i(x',k',j'|x,k,j) = p_j(x'|x)p_{i+1}^{k'} \quad (5)$$

where $x' \in \{z, z+l, \dots, (N_X\text{-}1)l, \zeta\}$; $k' \in \{k, k+1\}$

Recall that the deterioration state of the machine at the next decision epoch depends only on its deterioration state at the current epoch decision and the technological generation of this machine, denoted by $p_j(x'|x)$; and $p_{i+1}^{k'}$ is the appearance probability of the next technological generation (k+1) at the next decision epoch (i+1) with $k' = k+1$ or inversely, it is the non-appearance probability with $k' = k$.

$$p_j(x'|x) = \int_{z'-z}^{z'-z+l} f_j(y)dy \quad (6)$$

with z, $z' \in \{0, l, 2l, 3l \dots (N_X\text{-}1)l\}$ and $f_j(y)$ is the probability density function of the deterioration process of the machine's generation j^{th} within the decision period τ. Similarly,

$$p_j(m|x) = \int_{\zeta-z}^{\infty} f_j(y)dy \quad (7)$$

4 NUMERICAL EXAMPLES

In this section, we present numerical examples to illustrate the performance of our model.

4.1 *Input parameters*

4.1.1 *The appearance probability of new technology*

We define the appearance probability of new technology k+1 at decision epoch i+1, given the latest available technology at decision epoch i is k, as a time increasing function:

$$p_{i+1}^{k+1} = (1-\delta\varepsilon^{i-k}) \quad (8)$$

δ is the factor that reflects the non-appearance probability of next generation (k+1) at next decision epoch (i+1) when the latest available technology at the current epoch i is k and $k \equiv i$. The smaller δ is, the greater appearance probability is. And ε is the factor characterized the increasing rate of the appearance probability of new technology over time; ie. if the technological generation k+1 is not appear at decision epoch i+1, then it can appear at the next decision epoch (i+2) with probability 1 - $\delta\varepsilon$ given the appearance probability of (k+1) at (i+1) is 1 - δ. We have: $\delta, \varepsilon \in [0, 1]$.

4.1.2 *Deterioration process*

We consider the machine whose degradation process is modeled by the Gamma distribution: Gamma processes are often used to model the equipment's degradation (Van der Weide et al. 2007, Van Noortwijk. 2009).

In any decision period, the increments of deterioration $X_i(i + 1) - X_i(i)$ are independent, identical, and follow the stationary Gamma distribution with shape parameter $\alpha_i\tau$ (recall that τ is length of a decision period) and scale parameter β. The probability density function of the deterioration process of the machine's generation j in decision period τ is:

$$f_j(x) = \frac{\beta^{\alpha_j\tau}}{\Gamma(\alpha_j\tau)} x^{\alpha_j\tau-1} e^{-\beta x} \quad (9)$$

where β is a constant and a discussion for the improvement of α_j is given in the next paragraph.

4.1.3 *Impacts of the technological evolution*

As we have assumed, the technological evolution aims to improve degradation characteristics, and specifically the expected degradation rate. In case of stationary gamma processes, this expected degradation rate is directly proportional to the shape parameter α_i. We model the impact of the technological evolution with the following decreasing exponential geometric function:

$$\alpha_j = ae^{-\kappa(j-1)} + b \quad (10)$$

where κ, a, b are constants; $j \geq 1$.

Due to technological development, the deterioration rate of the machine is improved. It is convergent to the critical value, b, but the deterioration could not be excluded. We choose arbitrarily κ, a, b such as values in Table 1.

Additionally, under technological evolution, the purchase price of a new machine is assumed to be decreasing over time and normally increasing over technological generation:

$$c_{i,k} = c_{1,1}v^{i-1}u^{k-1} \quad (11)$$

where $c_{1,1}$ is the purchase price the first of technological generation at the first decision epoch; v is a constant, characterizing the decrease of purchase price over time ($v \leq 1$) and u is constant, characterizing the change of the purchase price over technological generation. We choose arbitrarily $c_{1,1}$, v, u such as values in Table 1, then $c_{i,k}$ is given in Table 2.

We assume the salvage value is a function of the current purchase price of this technology at this decision epoch, and the Mean Residual Lifetime (*MRL*).

According to the degradation assumptions, if x is the observed state, we define the $MRL(x)$ as the expected number of decision epoch from the current decision epoch until the failure. In case of stationary gamma processes, the mean deterioration rate on a decision epoch is constant and equals to $\alpha_j\tau/\beta$. Hence, we have:

$$MRL(x) = \frac{\beta}{\alpha_j \tau}(\zeta - x). \tag{12}$$

Then, we propose the following function for the salvage value, $\forall\, x \in [0, \zeta]$

$$b_{i,j}(x) = hc_{i,j}[1 - \exp(-rMRL(x))]$$

$$= hc_{i,j}[1 - \exp(-r\frac{\beta}{\alpha_j \tau}(\varsigma - x))] \tag{13}$$

h, r are constant.

4.1.4 *The profit and maintenance cost function*

We know that the machine will operate less efficiently when its deterioration state is greater. Therefore, the expected accrued profit function in a decision period τ is decreasing by deterioration state and the greater the deterioration state is, the faster the decreases of the profit function is. To reflect this nature, we use a decreasing concave function of deterioration state x to characterize the accrued profit.

$$g(x) = g_0 - r_g \exp(r\frac{\beta}{\alpha_1 \tau}x) \tag{14}$$

$\forall\, x \in [0, \zeta]$; g_0, r_g are constant.

On the contrary, the greater the deterioration state is, the faster the increase of the maintenance cost function is. Therefore, we use an increasing convex maintenance cost function.

$$c_M(x) = c_0 + r_c \exp(r\frac{\beta}{\alpha_1 \tau}x) \tag{15}$$

$\forall\, x \in [0, \zeta]$; c_0, r_c are constant.

Table 1. The input parameters for the Example 1

Appearance probability	δ	ε			
	0.8	0.96			
Profit & Discount factor	g_0	r_g	λ		
	213.2	1.2	0.8		
Maintenance & Failure threshold	d	c_0	r_C	ζ	
	1.4	3.322	0.178	20	
Deterioration process	β	a	b	κ	N_X
	2.22	3	0.4	0.4	100
Salvage value & Purchase price	h	r	$c_{1,1}$	v	u
	0.8	0.4	100	0.98	1.05

Table 2. The purchase price for the Example 1 of the technology k at decision epoch i, N = 5.

i	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$	$c_{i,4}$	$c_{i,5}$
1	100				
2	98	102.9			
3	96.04	100.84	105.88		
4	94.12	98.83	103.77	108.95	
5	92.24	96.85	101.69	106.78	112.11

4.2 *Analysis from numerical experiments*

4.2.1 *Basic properties of the optimal policy.*

The optimal policy for the Example 1 is given in Table 3. For each decision epoch i, with the used technology j, given the generation k is the latest available technology, the decision matrix defines the optimal maintenance decision according to the current deterioration x: Do nothing if $x \in [x_1, x_2)$, maintain if $x \in [x_2, x_3)$ and replace with the new technology if $x \in [x_3, x_4]$.

Table 3. The optimal policy in Example 1, N = 5

i	k, j	Do Nothing $[x_1,\ x_2)$	Maintenance $[x_2,\ x_3)$	Replacement $[x_3,\ x_4]$
1	1, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]
2	1, 1	[0, 2.3)	[2.3, 7.7)	[7.7, 20]
	2, 1	[0, 2.3)	[2.3, 8.3)	[8.3, 20]
	2, 2	[0, 2.9)	[2.9 , 7.7)	[7.7, 20]
3	1, 1	[0, 3.7)	[3.7, 8.3)	[8.3, 20]
	2, 1	[0, 3.7)	[3.7, 9.1)	[9.1, 20]
	2, 2	[0, 4.3)	[4.3, 8.3)	[8.3, 20]
	3, 1	[0, 3.7)	[3.7, 9.9)	[9.9, 20]
	3, 2	[0, 4.3)	[4.3, 9.1)	[9.1, 20]
	3, 3	[0, 4.5)	[4.5, 8.5)	[8.5, 20]
4	1, 1	[0, 5.9)	[5.9, 9.5)	[9.5, 20]
	2, 1	[0, 5.9)	[5.9, 10.5)	[10.5, 20]
	2, 2	[0, 6.1)	[6.1, 9.3)	[9.3, 20]
	3, 1	[0, 5.9)	[5.9, 11.3)	[11.3, 20]
	3, 2	[0, 6.1)	[6.1, 10.3)	[10.3, 20]
	3, 3	[0, 6.3)	[6.3, 9.3)	[9.3, 20]
	4, 1	[0, 5.9)	[5.9, 11.9)	[11.9, 20]
	4, 2	[0, 6.1)	[6.1, 11.1)	[11.1, 20]
	4, 3	[0, 6.3)	[6.3, 10.3)	[10.3, 20]
	4, 4	[0, 6.5)	[6.5, 9.7)	[9.7, 20]
5	1, 1	[0, 10.1)	[10.1, 12.9)	[12.9, 20]
	2, 1	[0, 10.1)	[10.1, 13.7)	[13.7, 20]
	2, 2	[0, 10.1)	[10.1, 11.7)	[11.7, 20]
	3, 1	[0, 10.1)	[10.1, 14.5)	[14.5, 20]
	3, 2	[0, 10.1)	[10.1, 12.9)	[12.9, 20]
	3, 3	[0, 10.1)	[10.1, 11.5)	[11.5, 20]
	4, 1	[0, 10.1)	[10.1, 15.3)	[15.3, 20]
	4, 2	[0, 10.1)	[10.1, 13.7)	[13.7, 20]
	4, 3	[0, 10.1)	[10.1, 12.5)	[12.5, 20]
	4, 4	[0, 10.1)	[10.1, 11.5)	[11.5, 20]
	5, 1	[0, 10.1)	[10.1, 15.9)	[15.9, 20]
	5, 2	[0, 10.1)	[10.1, 14.5)	[14.5, 20]
	5, 3	[0, 10.1)	[10.1, 13.3)	[13.3, 20]
	5, 4	[0, 10.1)	[10.1, 12.5)	[12.5, 20]

5, 5	[0, 10.1)	[10.1, 11.7)	[11.7, 20]

We find that the optimal policy for Example 1, given in Table 3 has some basic properties:

1) The maintenance threshold (x_2), i.e. the first time where the optimal policy prescribes to maintain across deterioration state x, depends only on the technological generation of the used machine j. Consider, for example, at decision epoch i = 3, the used technology j = 1, for which the optimal policy prescribes maintenance from the deterioration state x_2 = 3.7 despite the latest available technology k = 1, 2, or 3. Moreover, the greater the used technology is, the higher the threshold is, because deterioration rate is improved under technological development. For example, at i = 3, k = 3, this threshold is: x_2 = 3.7; 4.3; 4.5 for the used technology j = 1, 2, 3 respectively.

2) The replacement threshold (x_3) is non-decreasing in the difference between the latest available technology and the used technology because the purchase price is increasing over technological generation. For example, at decision epoch i = 3, when the latest available technology is k = 3, the replacement threshold is 9.9; 9.1; 8.5 for the technology used is j = 1, 2, 3 respectively. Certainly, this threshold depends also on used technological generation (j). It is non-decreasing in the used technology j. For example, at the decision epoch i = 3, when the used technology (j) is also the latest technology available (k), the replacement threshold is 8.3; 8.3; 8.5 for j = k = 1, 2, 3 respectively.

These properties are maintained even if the finite horizon N is large enough. Consider, Example 2 with the input parameters as Example 1: the optimal policy for the first three decision epochs in planning horizon N = 20 is given in Table 4.

Table 4. The optimal policy for the first three decision epochs in planning horizon N = 20.

i	k, j	Do Nothing $[x_1, x_2)$	Maintenance $[x_2, x_3)$	Replacement $[x_3, x_4]$
1	1, 1	[0, 1.5)	[1.5, 7.3)	[7.3, 20]
2	1, 1	[0, 1.5)	[1.5, 7.1)	[7.1, 20]
	2, 1	[0, 1.5)	[1.5, 7.3)	[7.3, 20]
	2, 2	[0, 1.7)	[1.7, 7.1)	[7.1, 20]
3	1, 1	[0, 1.5)	[1.5, 7.1)	[7.1, 20]
	2, 1	[0, 1.5)	[1.5 7.1)	[7.1, 20]
	2, 2	[0, 1.7)	[1.7, 7.1)	[7.1, 20]
	3, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]
	3, 2	[0, 1.7)	[1.7, 7.5)	[7.5, 20]
	3, 3	[0, 1.9)	[1.9, 7.3)	[7.3, 20]

4.2.2 *Influence of the technological improvement parameter on the optimal policy*

Recall that technological development is characterized by the improvement of the deterioration rate and the change of the purchase price $c_{i,k}$. Now, we consider the influence of these parameters on the optimal maintenance-replacement policy.

Note that the characterization of purchase price is represented by equation: $c_{i,k} = c_{1,1}v^{i-1}u^{k-1}$ where u is parameter that reflects directly the change of purchase price under technological development. When u > 1, the purchase price is increasing in technological generation, inversely, u < 1 this is the case where the technological improvement contributes to reduce the purchase price, and u = 1 is the case where the technological change does not influent on the purchase price.

As illustrated by the numerical examples in planning horizon N = 20, with u = 0.95, 1 and 1.05, consider the first three decision epochs (Table 5), we find that at the first decision epoch, the smaller u is, the higher the replacement threshold (x_3) is. x_3 = 7.7; 7.5; 7.3 respectively. The firm tends to keep the machine used for waiting the appearance of new technology. In the case where the new technology was available on the market, the firm tends to replace earlier when u is smaller. For example, at decision epoch i = 2, given j = 1 and k = 2, the replacement threshold is 7.3, 6.9, 6.5 for u = 1.05, 1, 0.95 respectively.

Specially, in the obsolete case, when the firms decide to replace early, the optimal policy can be non-monotone with respect to the DN and DM across deterioration x for given k, j. For example, with u = 0.95, at decision epoch i = 2, for k = 2, j = 1, the optimal policy prescribes do nothing until x = 2.5, maintain from x = 2.5 to 4.7 and then do nothing again at x = 4.7 until 6.5 (Table 6).

Table 5. The replacement threshold for the first three decision epochs in planning horizon N = 20 with u = 0.95; 1; 1.05

i	k, j	u = 0.95	u = 1	u = 1.05
1	1, 1	7.7	7.5	7.3
2	1, 1	7.7	7.5	7.1
	2, 1	6.5	6.9	7.3
	2, 2	7.1	7.1	7.1
3	1, 1	7.7	7.5	7.1
	2, 1	6.5	6.9	7.1
	2, 2	7.1	7.1	7.1
	3, 1	5.5	6.5	7.5
	3, 2	6.1	6.5	7.5
	3, 3	6.7	6.9	7.3

Table 6. The optimal policy for the first three decision epochs in the planning horizon N = 20 with u = 0.95

i	k, j	Do Nothing	Maintenance	Replacement
1	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]
2	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]

	2, 1	[0, 2.5)	[2.5, 4.7)	
		[4.7, 6.5)		[6.5, 20]
	2, 2	[0, 1.9)	[1.9, 7.1)	[7.1, 20]
3	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]
	2, 1	[0, 3.1)	[3.1, 4.1)	
		[4.1, 6.5)		[6.5, 20]
	2, 2	[0, 1.9)	[1.9, 6.9)	
		[6.9, 7.1)		[7.1, 20]
	3, 1	[0, 5.5)	-------------	[5.5, 20]
	3, 2	[0, 6.1)	-------------	[6. 1, 20]
	3, 3	[0, 2.1)	[2.1, 6.7)	[6.7, 20]

Now, we will consider how the improvement of the deterioration rate influences the optimal policy. Recall that the shape parameter of stationary Gamma function of deterioration process is represented by the decreasing exponential geometric function (equation 10); where κ characterizes directly the improvement of the deterioration rate.

We implement numerical examples in planning horizon $N = 20$ with $\kappa = 0.4$, 1, 1.5 and obtain the results in Table 7. We find that when $j = k$, the replacement threshold is non decreasing in κ. Specially, at the first decision epoch, the replacement threshold is increasing in κ, because the firms tend to replace later for waiting the new technology when the improvement of deterioration rate is more efficient (κ is increasing). Consider, at $i = 1$, the shape parameter of the first technological generation is the same as in the case where $\kappa = 0.4$, 1, 1.5, then, the replacement threshold is 7.3, 7.5, 7.7, respectively. The case where the obsolete problem appears ($j < k$) is more complex. The replacement threshold is non-monotonic in κ. For example, at $i = 3$, the replacement threshold is decreasing in κ, for ($k = 2, j = 1$) or for ($k = 3, j = 1$), but it is increasing in κ for ($k = 3, j = 2$).

Moreover, this parameter (κ) influences also on the maintenance policy such as: the increase of the maintenance threshold (x_2) in $j > 1$ and the appearance of the non-monotone property with respect to the DN and DM across deterioration x for given k, j. As illustrated by Table 8, with $\kappa = 1.5$, at decision epoch $i = 2$, for $k = 2, j = 1$, the optimal policy prescribes do nothing until $x_2 = 1.5$, maintain from $x = 1.5$ to 5.9 and then do nothing again from $x = 5.9$ to 6.3.

Table 7. The dependence of the replacement threshold on κ in planning horizon $N = 20$

i	k, j	$\kappa = 0.4$	$\kappa = 1$	$\kappa = 1.5$
1	1, 1	7.3	7.5	7.7
2	1, 1	7.1	7.5	7.5
	2, 1	7.3	6.5	6.3
	2, 2	7.1	7.3	7.3
3	1, 1	7.1	7.3	7.5
	2, 1	7.1	6.5	6.3
	2, 2	7.1	7.1	7.3
	3, 1	7.5	6.9	6.7
	3, 2	7.5	7.7	7.9
	3, 3	7.3	7.5	7.5

Table 8. The optimal policy for the first three decision epochs in the planning horizon $N = 20$ with $\kappa = 1.5$

i	*k, j*	Do Nothing	Maintenance	Replacement
1	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]
2	1, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]
	2, 1	[0, 1.5)	[1.5, 5.9)	
		[5.9, 6.3)		[6.3, 20]
	2, 2	[0, 2.1)	[2.1, 7.3)	[7.3, 20]
3	1, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]
	2, 1	[0, 1.5)	[1.5, 5.9)	
		[5.9, 6.3)		[6.3, 20]
	2, 2	[0, 2.1)	[2.1, 7.3)	[7.3, 20]
	3, 1	[0, 1.5)	[1.5, 6.7)	[6.7, 20]
	3, 2	[0, 2.1)	[2.1, 7.9)	[7.9, 20]
	3, 3	[0, 2.1)	[2.1, 7.5)	[7.5, 20]

5 CONCLUSION

In this paper, we proposed a model that allows us to consider both the investment and the maintenance problem of the stochastic deterioration system under the technological development. It determines the maintenance strategy from the operator's point of view, based on parametric performance of system. In addition, it allows the manager to take into account the necessary information of technology change to decide the best time for replacement investment of equipment as well as to consider the impact of technological evolution on the maintenance strategies.

We have considered a lot of assumptions and parameters in our model to tackle the complexity of the decision environment for the maintenance managers. We have assumed that technological evolution is stochastic and the impact of a new technology can be measured not only from an economical point of view but also on the system deterioration performance. This high number of parameters is also due to our choice of integrating a quite well-advanced maintenance strategy (condition-based repair and replacement policy) to ensure the "local" optimality, i.e. independently on the technical change opportunity, in the strategic decision context.

We then used stochastic dynamic programming (i.e., discrete non-stationary Markov decision process) to solve for the optimal maintenance and replacement policy of the equipment as a function of performance and cost. And finally, we presented numerical examples to illustrate performance of our model and to consider the influence of the parameters characterized the technological development on the optimal maintenance-replacement policy.

Some proposed assumptions can be seen as limitations of our model. The uncertainty in the technological evolution, e.g., is just considered in the time of appearance of a new generation but the associated purchase cost and the deterioration improvement are deterministic. In fact, these can be stochastic and difficult to capture.

The future work could reflect the stochastic characterization of these parameters. Furthermore, the stochastic efficiency of the imperfect maintenance action could also be included in our model or the conception of technology horizon *N* such that the initial optimal decision would be invariant even if more than *N* technologies wear to appear in future, could be consider .

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